Reprint from:
D. Algom, D. Zakay, E. Chajut, S. Shaki, Y. Mama, V. Shakuf (eds) *Fechner Day 2011*.
Ra’anana, Israel: The International Society for Psychophysics; pp. 47–52.

FILLED SPACE EXPANSION: CONSTANTS, VARIANTS AND DETERMINANTS OF THE OPPEL–KUNDT PHENOMENON

Jiří Wackermann
Dept. of Empirical and Analytical Psychophysics
Institute for Frontier Areas of Psychology and Mental Health
Freiburg i. Br., Germany
<jw@igpp.de>

Abstract

The Oppel–Kundt (OK) phenomenon, also known as 'illusion of filled space', provides a paradigmatic example of an illusory distortion of the visual field metric. Studies using various forms of the OK figure reveal non-monotonic functional dependences of the effect on the extent, density and orientation of the filling elements. The importance of these findings, and their possible consequences for an interactionist theory of the OK phenomenon, are discussed.

A path in the visual field, which is subdivided into a number of shorter segments, appears longer than an undivided path of the same length. This phenomenon, belonging to a broad class of 'geometric–optical illusions' (GOI),[1] is traditionally called the 'Oppel–Kundt illusion' (OKI),[2] but it has been also dubbed the 'illusion of interrupted extent'[3] or the 'illusion of filled space'.[4] These variations of nomenclature indicate an ambiguity concerning the main determinant of the phenomenon, which is also reflected in the various graphic presentations of the phenomenon (Fig. 1).[5]

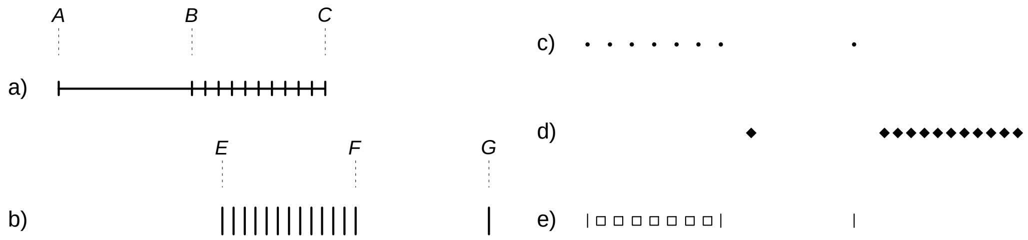

Figure 1. Oppel–Kundt phenomenon. (a) The distance between markers B, C is perceived as greater than that between A, B; (b) the distance between markers E, F is perceived as greater than that between F, G. The same effect is demonstrated by arrays of simple dots (c) or spatially extended elements (d). In (e), the apparent distance between the vertical strokes is affected by interpolated elements of a different appearance.

Most authors, following Oppel's original report,[6] referred to the 'division' or 'interruption' of a given path as the cause of the illusory effect. However, to perceive a spatial extent as divided, there must be other visible elements to mark the points of division (usually short perpendicular strokes: Fig. 1a). In fact, these additional elements are more important than a visual realization of the subdivided length itself, as evidenced by those variants of the Oppel–Kundt (OK) figure, in which the empty space between two delimiting marks is filled by an array of 'expletive' elements (Fig. 1b–e).[7] Is it their action of 'dividers' or 'fillers' that causes the illusory effect? This question is not a mere play of words, as will be seen shortly.

Oppel's observation was based on subjects' drawing squares on sheets of paper with a pre-printed raster of equispaced lines (Fig. 2a). A simple variation yields well-known Helmholtz' squares (Fig. 2b) or Botti's figures, showing an illusory expansion in a direction perpendicular to the filling lines. These relations to other, genuinely two-dimensional GOIs suggest that the 'classic' one-dimensional form of the OK phenomenon is a rather arbitrary abstraction.

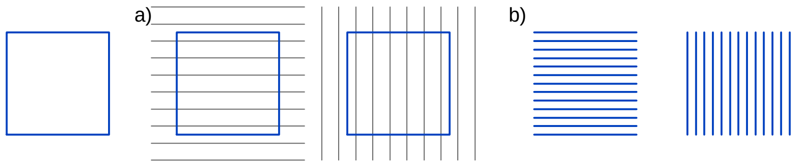

Figure 2. Square figure (a) drawn over a raster of parallel lines (Oppel), and (b) filled with a raster of parallel lines (Helmholtz). In both cases, the square form appears slightly elongated in the direction orthogonal to the raster lines.

Experimental data

Here we report results from four experimental series conducted in our laboratory. We use, as most researchers do,[8] one-dimensional arrangements of elements,[9] and a standard–variable adjustment method. The visual elements are short line segments of one p. e. width,[10] drawn black on a neutral white background. The subject has to position a movable marker V to make the perceived distance $\overline{VS_0}$ equal to $\overline{S_0 S_1}$ ('distance matching' task: Fig. 3a). The magnitude of the effect is evaluated as a relative deviation of the subject's response $v = \overline{VS_0}$ from the geometrically correct response $s = \overline{S_0 S_1}$, that is, $r = (v - s)/s$. The variable stimulus parameters are the number of the expletive elements n and their height h (Fig. 3b). The height of the delimiting marks h_{del} and the distance s are kept constant within a current experimental series.

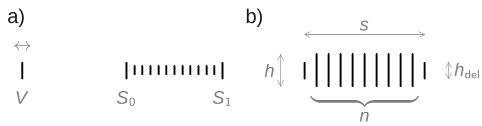

Figure 3. (a) Distance matching task. (b) Stimulus parameters: n = number of vertical strokes of height h; s = spatial extent, marked by strokes of height h_{del}.

Effects of vertical extent (Studies 1 and 2)

In the first study[11] we explored effects of the vertical extent $h = 0, 5, 15, 45$ (where $h = 0$ indicates the control condition, i. e., the space between S_0 and S_1 left empty), with constant settings $n = 20$, $s = 168$, $h_{\mathrm{del}} = 15$. The results are shown in Fig. 4a. Expectedly, we found no effect in the control condition, a significant illusory expansion for $h = 5$ and $h = 15$, and a drop in the effect to zero for $h = 45$. The maximum effect, $\overline{r} \approx 0.15$, was observed at $h = h_{\mathrm{del}}$.

In a follow-up study,[12] we examined these effects on a finer scale and in a wider variation range of $h = 0, 1, 3, 7, 15, 31, 63$; in *post hoc* series of experiments we attempted to identify individual loci of maximal effect on a finer scale. The results are shown in Fig. 4b. The maximum effect, $\overline{r} \approx 0.13$, was again observed at $h = h_{\mathrm{del}}$ (although the average of individual maxima was slightly shifted toward $h \approx 17$), followed by a rapid decrease to zero.

Effects of numerosity (Study 3)

In this study[13] we examined the effect as a function of the number n, varied in a wide range from $n = 2$ to 55, using constant $s = 168$, $h = h_{\mathrm{del}} = 15$. (Again, $n = 0$ indicates the 'empty space' control condition.) The results are shown in Fig. 4c. The maximum effect, $\overline{r} \approx 0.15$, was found for $n = 11$ and $n = 13$, followed by a plateau at $20 \leq n \leq 27$ and a slow decrease for more dense fillings; these findings are in generally good agreement with the literature.[14]

Importantly, the dependence of the average effect $\bar{r}$ on n as well as on h could be modeled by the same 2-parametric functional form,[15] which was helpful for smooth data interpolation and final estimates of the loci of maximal effect.

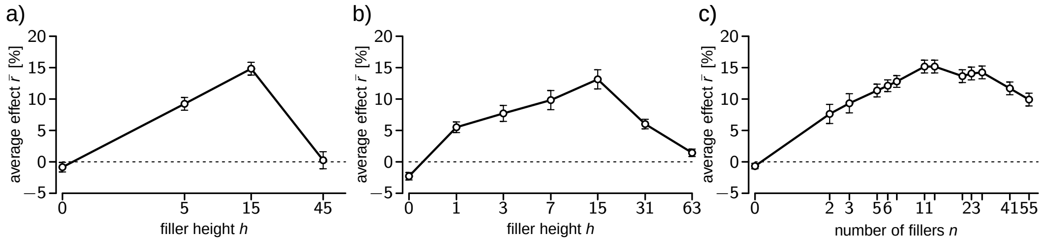

Figure 4. Results of studies 1–3. Average effects are plotted as functions of the respective control parameter, h or n (logarithmic scales). 0 = control task (empty space between S_0, S_1). Error bars indicate probable errors of the grand means.

Effects of fillers shape (Study 4)[16]

In the studies reported above, the total area filled by the vertical strokes between the two fixed delimiters S_0, S_1 co-varied linearly with the stimulus parameters h (for $h \leq h_{\mathrm{del}}$) or n, respectively. This observation suggested a question: Is the observed effect simply a function of the relative coverage of the estimated space?

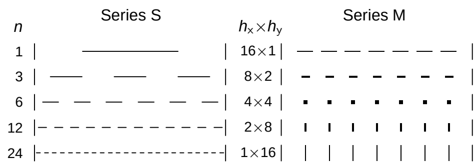

Figure 5. Stimuli used in study 4. Series S: successive splitting of a line of length 96 p. e., relative coverage 3.1%. Series M: morphing of rectangular elements of a constant area $h_x \times h_y = 16$ sq. p. e., relative coverage 3.6%.

To address this question, two subsets of stimuli were used in the next study, each subset consisting of five variants (Fig. 5). In the first subset (S), a variable number of equispaced horizontal line segments were used as the fillers, while the total sum of their lengths was constant. In the second subset (M), seven equispaced little rectangles of varying heigth-to-width ratio but a constant area were used as the fillers. Therefore, the relative coverage of the space between the delimiters with the expletive material was constant within each subset. The same apparatus and procedures were used as in studies 1–3, with constant settings $s = 192$ and $h_{\mathrm{del}} = 16$.

If the illusory expansion were dependent solely on the relative filling of the delimited area with the 'optical matter', then the effect would be expectedly positive but invariant within each one sub-series of experiments. This, however, was not the case, as seen in Fig. 6.

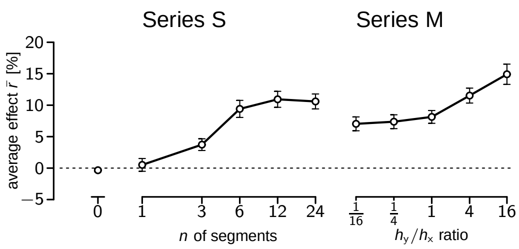

Figure 6. Results of study 4. Average effects are plotted as functions of the respective control parameter, n (number of line segments), or h_y/h_x ratio of filler sides (both on logarithmic scales). 0 = control task (empty space between S_0, S_1). Error bars indicate probable errors of the grand means.

In the S sub-series, the presence of one compact line segment had no significant effect; splitting up the line to $n = 3$ and 6 segments, a monotonically increasing effect was observed, which saturated at $\bar{r} \approx 0.105$ for the finest filling with $n = 12$ and 24. In the M sub-series ($n = 7$), a moderate effect $\bar{r} \approx 0.07$ was observed, which remained almost invariant for $h_y/h_x \leq 1$ and then steeply increased to $\bar{r} = 0.15$, where the fillers become identical to vertical strokes used in the previous studies.

Discussion and concluding remarks

The OKI turns out to be no less intriguing than other, more popular and more intensely studied GOIs. Probably the most interesting aspect of the phenomenon of our interest is the non-additivity of length in the visual field; symbolically,

$$\ell(P,R) < \ell(P,Q_1) + \cdots + \ell(Q_n,R)\,,$$

where $P, Q_1, \ldots, Q_n, R$ are points on a linear path, and ℓ is a function assigning a 'visual extent' to a segment of the path. It is this striking violation of additivity which makes *prima faciæ* the very existence of a well-defined metric in the visual field problematic.

Another striking feature is the non-monotonic dependence of the effect magnitude on the two stimulus variables under study, n and h. This effect observed w. r. t. the number of expletive elements is well-known and *per se* not surprising.[17] However, a similar non-monotonic variation w. r. t. the extent of the fillers in the direction orthogonal to the 'axis of the effect' was found, which interacts with the effect of n and may eventually cancel it. This presents a serious problem for a theory of the OK phenomenon. What, then, is the proper determinant of the effect?

Evidently none of the examined factors can be identified as the main or major determinant; rather, the results suggest an intimate interplay between the numerosity and the appearance of the contextual elements. The picture is further complicated by the spatial anisotropy of their action, indicated by results of study 4: the expansion/contraction effect acts predominantly in the direction orthogonal to the dominant edges of the expletive elements. A neurophysiologist may assume a rôle of 'hardwired' direction-specific detectors in the visual system; while a psychophysicist will search for a suitable form of a potential function modeling repulsive interactions between visual elements. If and how these approaches can be combined in a viable theory, remains to be seen.

Certainly the mysterious charm of the OKI consists in its elementary appearance. In standard presentation forms, the context and the target elements are of the same shape: there is only a linear array of strokes or dots. There are no intersections with other elements, no angles, no alleged 'depth cues', therefore no 'perspectival interpretation'. The OKI demonstrates mutual interactions between elements in the visual field in their purest form, but also in their puzzling complexity. Some factual knowledge about this phenomenon has been gathered, but its understanding is still insufficient. This situation not only leaves space for further research; it opens new spaces for more experimental and modeling work.

Acknowledgements: Thanks to Kristina Kastner and Oksana Gutina for conducting the experiments and for technical assistance. The author also wishes to thank Erik C. Banks and Werner Ehm for discussions and comments on earlier versions of the manuscript.

Notes

[1] A term coined by Oppel (1855).

[2] Oppel (1861) is credited with being the first to *describe* the 'interrupted extent' illusion, though Lotze (1852) mentioned the same phenomenon about a decade earlier (cf. Westheimer, 2008, p. 2138). Kundt (1863) was the first to *measure* the illusory effect in controlled experiments, using the reproduction method; he also proposed, almost simultaneously with Hering (1861), a geometric theory of the effect, which was soon disqualified by Aubert's (1865) measurements. To acknowledge all those early contributions, we should speak about (Lotze–)Oppel–Hering–Kundt–Aubert illusion. For convenience we prefer the traditional term Oppel–Kundt illusion.

[3] Sanford (1903), p. 233ff; Luckiesh (1922), p. 48ff; Boring (1942), p. 239.

[4] Lewis (1912); Obonai (1933); Robinson (1998), p. 49f.

[5] Fig. 1a is the 'classic' illustration of the OKI found in early publications (e. g. Helmholtz, 1867; Wundt, 1898). The variants in Fig. 1b,c are presented by most modern authors (e. g. Coren and Girgus, 1978; Robinson, 1998). However, none of these figures was published by Oppel himself (cf. Vicario, 2008).

[6] "Eine weitere hier zu erwähnende Beobachtung bezieht sich auf die vergleichende Schätzung getheilter Linien oder Flächen gegen ungetheilte und ununterbrochene. [...] [S]o läßt überhaupt die mehrfache Theilung einer Fläche oder einer Dimension dieselbe leicht größer erscheinen als eine [...] ebenso große ungetheilte Fläche oder Linie." (Oppel, 1861, p. 35) — "Yet another observation to be mentioned here is about judgment of divided lines or areas as compared to undivided and uninterrupted ones. [...] Generally, a multiple division of an area, or of a [linear] dimension, makes the latter appear slightly larger than [...] an undivided area of the same size." (Translation J. W.)

[7] The term 'expletive element' was introduced by Wackermann and Kastner (2009) to avoid presumptions concerning the 'dividing' or 'filling' action.

[8] This is no place, and we have no intention, to review results of 150 years of research. The most extensive experimental investigation of the OKI up to present days is probably the study by Spiegel (1937). In the recent decade, systematic studies were contributed particularly by the Lithuanian research group (Bulatov et al., 1997; Bulatov and Bertulis, 1999; Bertulis et al., 2009).

[9] In early experiments, using mechanical (Kundt, 1863; Aubert, 1865) or opto-mechanical (Spiegel, 1937) devices, the restriction to a single dimension was enforced by construction principles of the apparatus. Modern computer-generated visual displays do not impose such principal limitation, but the tradition of studying the effect as a one-dimensional 'illusion of extent' continues.

[10] Lengths are specified in 'picture elements' (p. e.) of the display device. In our experimental set-up, one p.e. in the display plane corresponded exactly to a visual angle of one minute of arc; hereafter we omit these units for the sake of convenience. See Wackermann and Kastner (2009) for technical details.

[11] Wackermann and Kastner (2009).

[12] Wackermann and Kastner (2010), Experiment 1.

[13] Wackermann and Kastner (2010), Experiment 2.

[14] Dependence of the effect on n was studied and its non-monotonic course was reported by several researchers (Knox, 1894; Spiegel, 1937; Bulatov et al., 1997; Bertulis et al., 2009). However, we had to determine this functional dependence specifically for our experimental arrangement and stimuli.

[15]
$$F(x) = x^{\alpha} \left(\frac{\alpha + \beta}{\alpha x + \beta} \right)^{\alpha+\beta}, \qquad \text{where } x \geq 0,\ \alpha, \beta > 0.$$

For a derivation see Appendix B in Wackermann and Kastner (2010).

[16] Wackermann (2011).

[17] Non-monotonic dependences of the illusory effects on the spatial density of contextual elements have also been noted for other GOIs, e. g. for the Hering figure (Holt-Hansen, 1961) or the Ehrenstein figure (Yoshino et al., 2009). Recently, Giora and Gori (2010) also reported non-monotonic dependencies of perceived area expansion on the density of (random or regular) filling patterns; this phenomenon can be conceived of as a 2-dimensional analog of the 'classic' OKI.

References

Aubert, H. (1865). *Physiologie der Netzhaut*. Breslau: Morgenstern.

Bertulis, A., Surkys, T., Bulatov, A., and Gutauskas, A. (2009). Three-part Oppel–Kundt illusory figure. *Medicina (Kaunas)*, *45*, 871–877.

Boring, E. G. (1942). *Sensation and Perception in the History of Experimental Psychology.* New York: Appleton–Century–Crofts.

Botti, L. (1906). Ein Beitrag zur Kenntnis der variabeln geometrisch-optisch Streckentäuschungen. *Archiv für die gesamte Psychologie*, *6*, 306–315.

Bulatov, A., Bertulis, A., and Mickienė, L. (1997). Geometrical illusions: study and modelling. *Biological Cybernetics 77*, 395–406.

Bulatov, A. and Bertulis, A. (1999). Distortions of length perception. *Biological Cybernetics 90*, 185–193.

Coren, S. and Girgus, J. S. (1978). *Seeing is Deceiving: The Psychology of Visual Illusions.* Hillsdale (NJ): Lawrence Erlbaum.

Giora, E. and Gori, S. (2010). The perceptual expansion of a filled area depends on textural characteristics. *Vision Research*, *50*, 2466–2475.

Helmholtz, H. (1867). *Handbuch der physiologischen Optik.* Leipzig: Voss.

Hering, E. (1861). *Beiträge zur Physiologie, Vol. 1*. Leipzig: Engelmann.

Holt-Hansen, K. (1961). Hering's illusion. *British Journal of Psychology*, *52*, 317–321.

Kundt, A. (1863). Untersuchungen über Augenmaass und optische Täuschungen. *Annalen der Physik und Chemie*, *196*, 118–158.

Knox, H. W. (1894). On the quantitative determination of an optical illusion. *American Journal of Psychology*, *6*, 413–420.

Lewis, E. O. (1912). The illusion of filled space. *British Journal of Psychology*, *5*, 36–50.

Lotze, R. H. (1852). *Medicinische Psychologie oder Physiologie der Seele.* Leipzig: Weidmann.

Luckiesh, M. (1922). *Visual Illusions, Their Causes, Characteristics and Applications*. New York: Van Nostrand.

Obonai, T. (1933). Contributions to the study of psychophysical induction: III. Experiments in the illusions of filled space. *Japanese Journal of Psychology*, *8*, 699–720.

Oppel, J. J. (1855). Ueber geometrisch-optische Täuschungen. *Jahresberichte des physikalischen Vereins zu Frankfurt 1854/1855*, 37–47.

Oppel, J. J. (1861). Ueber geometrisch-optische Täuschungen (Zweite Nachlese). *Jahresberichte des physikalischen Vereins zu Frankfurt 1860/1861*, 26–37.

Robinson, J. O. (1998). *The Psychology of Visual Illusion.* 2nd ed., Mineola (NY): Dover.

Sanford, E. C. (1903). *A Course in Experimental Psychology I: Sensation and Perception*. Boston: Heath.

Spiegel, H.-G. (1937). Über den Einfluß des Zwischenfeldes auf gesehene Abstände. *Psychologische Forschung*, *21*, 327–383.

Vicario, G. B. (2008). Optical–geometrical illusions: the nomenclature. *Gestalt Theory*, *30*, 168–180.

Wackermann, J. and Kastner, K. (2009). Paradoxical form of filled/empty optical illusion. *Acta Neurobiologiae Experimentalis*, *69*, 560–563.

Wackermann, J. and Kastner, K. (2010). Determinants of filled/empty optical illusion: search for the locus of maximal effect. *Acta Neurobiologiae Experimentalis*, *70*, 423–434.

Wackermann, J. (2011). Determinants of filled/empty optical illusion: differential effects of patterning. *Acta Neurobiologiae Experimentalis*, submitted.

Westheimer, G. (2008). Illusions in the spatial sense of the eye: Geometrical–optical illusions and the neural representation of space. *Vision Research*, *48*, 2128–2142.

Wundt, W. (1898). Die geometrisch–optischen Täuschungen. *Abhandlungen der mathematisch-physischen Classe der Königlichen Sächsischen Gesellschaft der Wissenschaften*, *24*, 55–178.

Yoshino, D., Kimura, A., and Noguchi, K. (2009). Visual illusion and aesthetic preference: some common stimulus properties. *Gestalt Theory*, *31*, 29–42.